HEAPS

Announcements

- PA03 checkpoint due tomorrow (05/31) at midnight. Submit to pa03-checkpoint assignment on gradescope
- PA03 due next Friday 06/08 at midnight. Submit to the pa03 assignment on gradescope
- Final exam on Wed 06/13 (8a -11a)
- Review session on Tuesday (06/05): first session 2:00p to 3:00p and the other from 3:00p to 4:00p

How is PA03 going?

A. Done
B. On track to finish
C. Having trouble with the checkpoint (design)
D. Just started
E. Haven’t started

Heaps: Supported Operations

	Min-Heaps	Max-Heap	BST (balanced)
• Insert :	O(log N)	O(log N)	O(log N)
• Min:	O(1)	—	O(log N)
• Delete Min:	O(log N)	—	"
• Max	—	O(1)	"
• Delete Max	—	O(log N)	"

Choose heap if you are doing repeated insert/delete/(min OR max) operations

Applications:

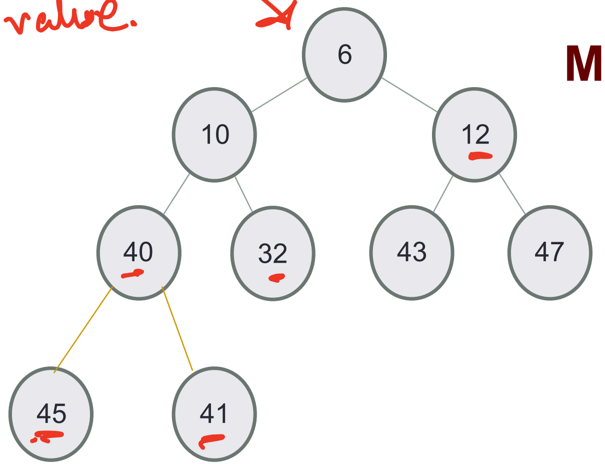
- Efficient sort
- Finding the median of a sequence of numbers
- Compression ~~codes~~ algorithms

Heaps as binary trees

- **Rooted binary tree that is as complete as possible**
- **In a min-Heap, each node satisfies the following heap property:**

key(x)<= key(children of x)

root holds the min value.



Min Heap with 9 nodes

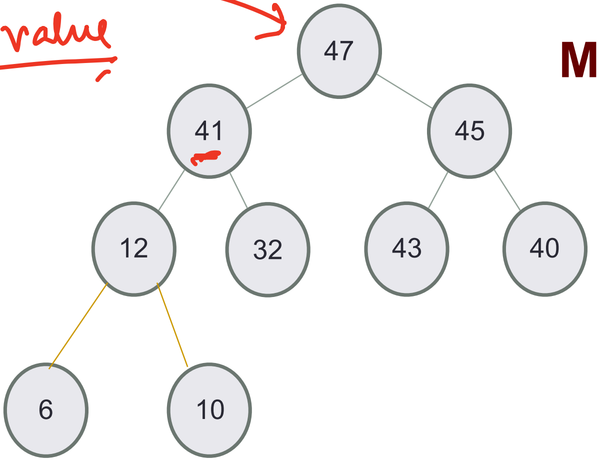
Where is the minimum element?

Heaps as binary trees

- **Rooted binary tree that is as complete as possible**
- **In a max-Heap, each node satisfies the following heap property:**

key(x)>= key(children of x)

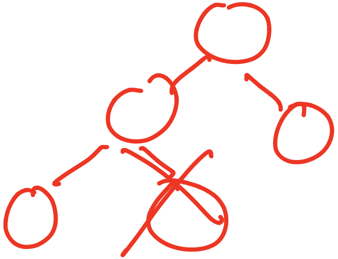
root holds the max value



Max Heap with 9 nodes

Level 0

Level

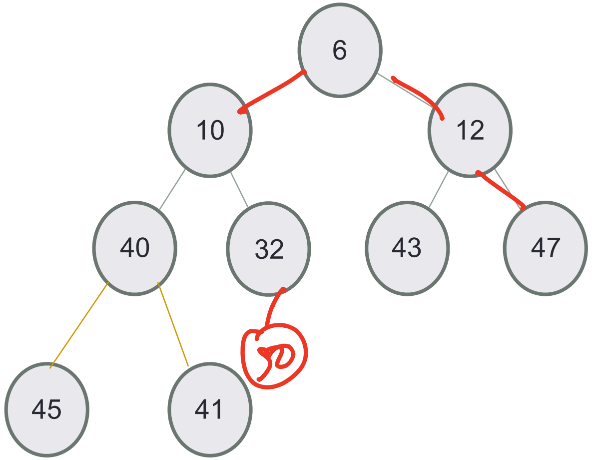


Where is the maximum element?

Identifying heaps

Starting with the following min Heap which of the following operations will result in something that is NOT a min Heap

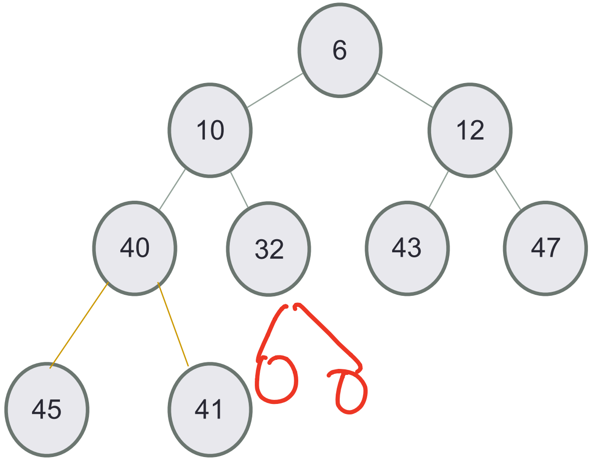
A. Swap the nodes 40 and 32
B. Swap the nodes 32 and 43
C. Swap the nodes 43 and 40
D. Insert 50 as the left child of 45
E. C&D



Structure: Complete binary tree

A heap is a complete binary tree: Each level is as full as possible.

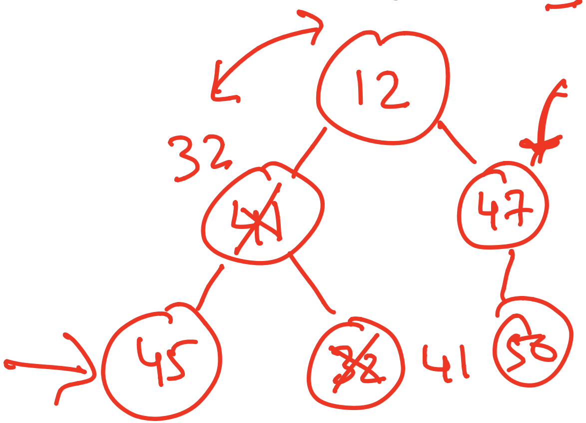
Nodes on the bottom level are as far left as possible



Insert into a heap

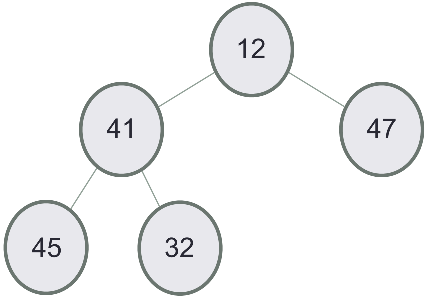
- Insert key(x) in the first open slot at the last level of tree (going from left to right)
- If the heap property is not violated - Done

Insert the elements {12, 41, 47, 45, 32} in a min-Heap



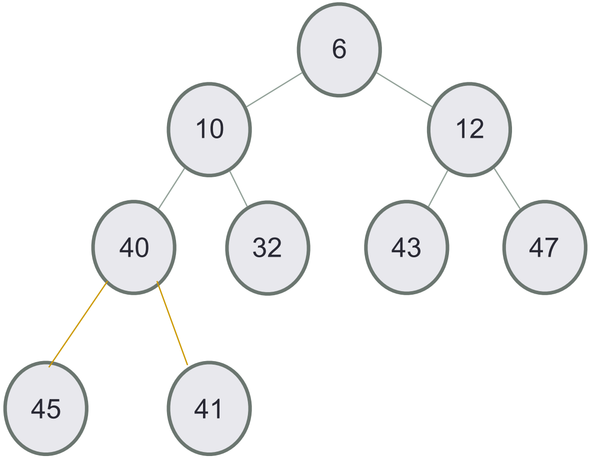
Insert 32 into a heap

- Insert key(x) in the first open slot at the last level of tree (going from left to right)
- If the heap property is not violated - Done
- Else: while(key(parent(x))>key(x)) swap the key(x) with key(parent(x))



Implementing heaps as arrays

Value	6	10	12	40	32	43	47	45	41	50
Index	0	1	2	3	4	5	6	7	8	10



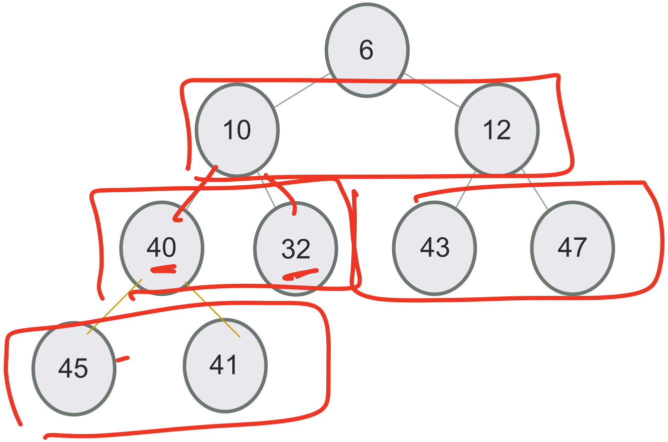
How is the array implementation of the heap useful?

- **More space efficient**
- **Accessing parent and children of a node is O(1)**
- **Easier to insert elements in the heap**

Conceptualize heaps as trees, implement as arrays

Value	6	10	12	40	32	43	47	45	41
Index	0	1	2	3	4	5	6	7	8

1, 2   3, 4   5, 6   7, 8



For a node at index i, what is the index of the left and right children?

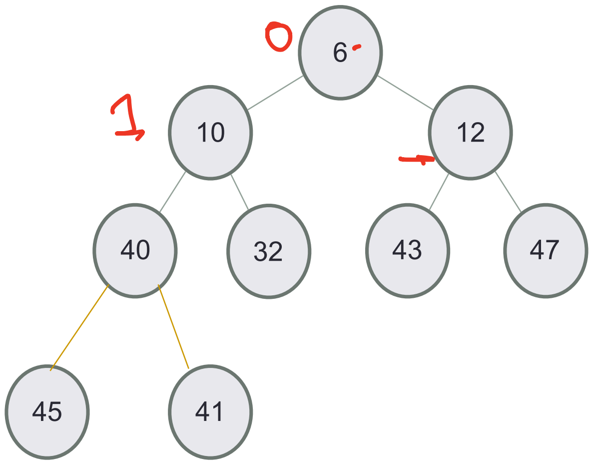
A. (2*i, 2*i+1)

B. (2*i+1, 2*i+2)

C. (log(i), log(i)+1)

D. None of the above

Conceptualize heaps as trees, implement as arrays



Value	Index	Index of parent	Index of children
6	0	-	1, 2
10	1	0	3, 4
12	2	0	5, 6
40	3	1	7, 8
32	4	1	
43	5	2	
47	6	2	
45	7	3	
41	8	3	

For a node at index i, index of the parent is:

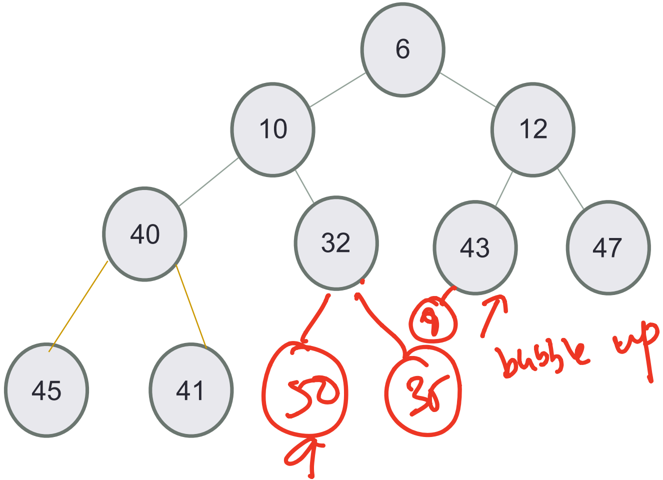
i/2 - 1, **if i is even**

$\boxed{\textbf{(i-1)/2,}}$ **if I is odd**

↳ this formula alone would work if we are using integer division

Insert 50, then 35

Value	6	10	~~12~~ 8	40	32	~~43~~ ~~8~~ 12	47	45	41	50	35	~~8~~ 43
Index	0	1	2	3	4	5	6	7	8	9	10	11



For a node at index i,

*** index of the parent is:**

i/2 - 1, if i is even

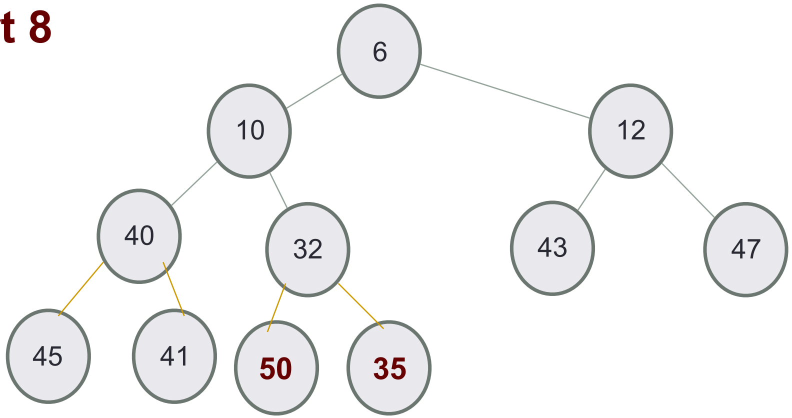
(i-1)/2, if i is odd

*** index of the children are (2i+1, 2i+2)**

Insert 8 into a heap

Value	6	10	12	40	32	43	47	45	41	50	35
Index	0	1	2	3	4	5	6	7	8	9	10

Insert 8



Which node is the parent of 8 after the insertion has been completed

A. Node 6

B. Node 12

C. None 43

D. None - Node 8 will be the root

For a node at index i,

*** index of the parent is:**

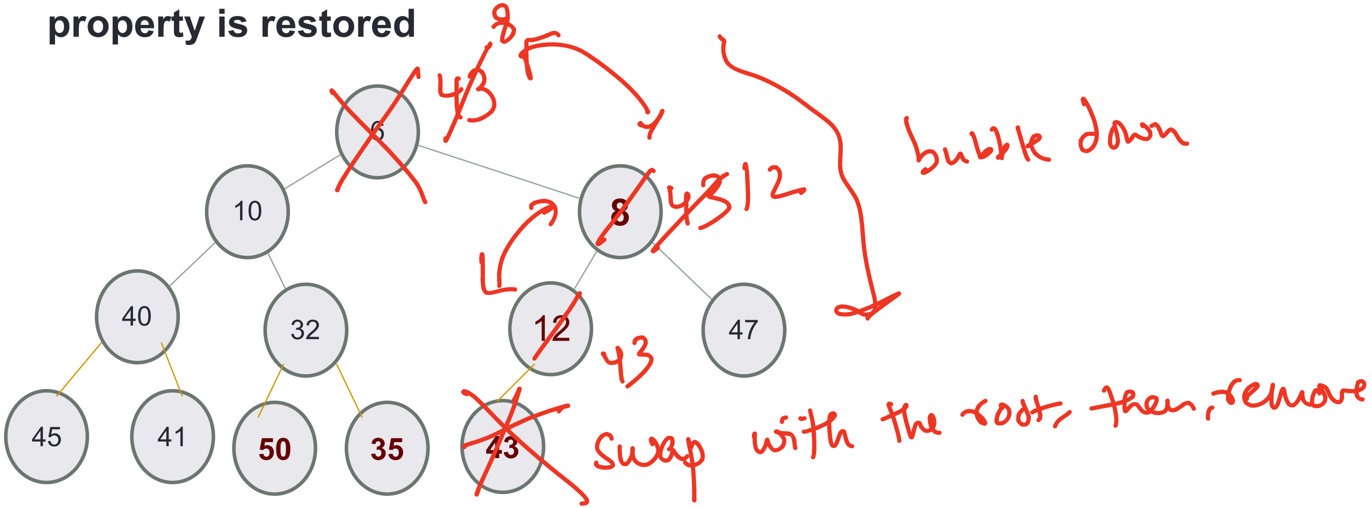
i/2 - 1, if i is even

(i-1)/2, if i is odd

*** index of the children are (2i+1, 2i+2)**

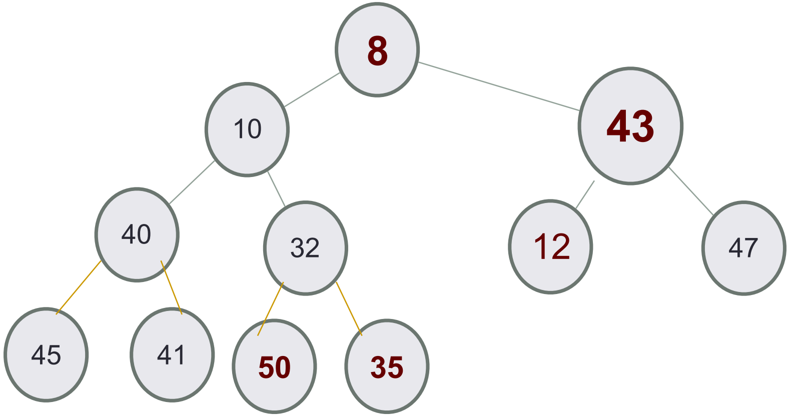
Delete min

- Replace the root with the rightmost node at the last level
- "Bubble down"- swap node with one of the children until the heap property is restored



Delete min

- **Delete the root:**
  - **Replace the root with the last node in the array**
  - **If heap property is violated - swap with the child that has the LOWEST key value, repeat until heap property is restored**



To fix the heap property on a delete BUBBLE DOWN!

Worst case: O(logN)

Applications

- Efficient sort
- Finding the median of a sequence of numbers
- Compression codes